Section 11.3 Surface Areas of Polyhedra

1. Solution key:
   Area of base = $4(5) = 20$ units$^2$
   Area of lateral face = $4(7) = 28$ units$^2$
   Area of lateral face = $5(7) = 35$ units$^2$
   Surface area = $2(20) + 2(28) + 2(35) = 166$ units$^2$

2. 
   Surface area = $2(2)(6) + 2(2)(3) + 2(3)(6)$
   $= 72$ ft$^2$

3. 
   Surface area = $2(7)(4) + 2(7)(7) + 2(4)(7)$
   $= 210$ m$^2$

4. 
   Surface area = $2(3)(4) + 2(3)(5) + 2(4)(5)$
   $= 94$ in.$^2$

5. 
   Surface area = $2wh + 2lh + 2wh$

Sample rubric: Assign 1 point to each correct net and 1 point to each correct surface area or formula. There is a total of 9 points.
3.  
\[ S = 2(2 \cdot 9) + 2(7 \cdot 9) + 2(2 \cdot 7) 
\quad = 36 + 126 + 28 
\quad = 190 \text{ in}^2 \]

5. (a) 
\[ S = 2(2 \cdot 9) + 2(7 \cdot 9) + 2(2 \cdot 7) 
\quad = 36 + 126 + 28 
\quad = 190 \text{ cm}^2 \]

(b) 
\[ S = 2(8 \cdot 5) + 2(3 \cdot 8) + 2(3 \cdot 5) 
\quad = 80 + 48 + 30 
\quad = 158 \text{ cm}^2 \]

7. 
\[ S = 6(6 \cdot 6) 
\quad = 216 \text{ ft}^2 \]

9. (a) 
\[ S = 5 \cdot 5 + 4 \cdot \frac{1}{2}(12 \cdot 5) 
\quad = 25 + 120 
\quad = 145 \text{ in}^2 \]

(b) 
\[ S = \frac{1}{2} \cdot 2.6 \cdot 3 + 3 \cdot \frac{1}{2}(7 \cdot 3) 
\quad = 3.9 + 31.5 
\quad = 35.4 \text{ ft}^2 \]

11. 
\[ S = 2 \cdot \frac{1}{2}(12 \cdot 9) + 17 \cdot 15 + 12 \cdot 17 + 17 \cdot 9 
\quad = 108 + 255 + 204 + 153 
\quad = 720 \text{ in}^2 \]
13. (a) \( S = 6s^2 = 6(4)^2 = 96 \text{ m}^2 \)
(b) \( S = s^2 + 4 \cdot \frac{1}{2}bh = (2)^2 + 4 \cdot \frac{1}{2}(2)(5.5) = 26 \text{ in.}^2 \)

15. (a) True; Sample answer: The net contains all the faces.
(b) False; Sample answer: The surface area of the cube will increase by a factor of 9 because the side length is squared when computing the surface area.

17. (a) Blue rectangular prism:
\[
S = 3 \cdot 5 + 2 \cdot 3 \cdot (7 - 3) + 2 \cdot 5 \cdot (7 - 3)
= 15 + 24 + 40
= 79 \text{ units}^2
\]
Yellow rectangular prism:
\[
S = (7 - 5) \cdot 3 + 2 \cdot 3 \cdot 3 + 3 \cdot 7 \cdot 3
= 6 + 18 + 63
= 87 \text{ units}^2
\]
The surface area of the solid is 
79 + 87 = 166 square units.
(b) Blue rectangular prism:
\[
S = 1.5 \cdot 3 + 2 \cdot 3 \cdot (8 - 4) + 2 \cdot 1.5 \cdot (8 - 4)
= 4.5 + 24 + 12
= 40.5 \text{ units}^2
\]
Pink rectangular prism:
\[
S = (4 - 1.5) \cdot 3 + 3 \cdot 3 \cdot 4 + 2 \cdot 4 \cdot 4
= 7.5 + 36 + 32
= 75.5 \text{ units}^2
\]
The surface area of the solid is 
40.5 + 75.5 = 116 square units.

19. 1. Understand the Problem Find the length of each edge of the cube.
2. Make a Plan Use the formula for the surface area of a cube and input all known values. Then solve for the missing value.
3. Solve the Problem The surface area is 216 square centimeters.
\[
S = 6s^2
216 = 6s^2
\frac{216}{6} = \frac{6s^2}{6}
36 = s^2
\]
Because \( s^2 = 36 \), the length of each edge of the cube is 6 centimeters.

21. The surface area of the cube-shaped package is \( 6(1.5)^2 = 13.5 \text{ square feet} \). This is the least amount of wrapping paper you need to cover the package.

23. The surface area of the walls, without the windows and the door, is \( 2 \cdot 8 \cdot 14 + 2 \cdot 8 \cdot 10 = 384 \text{ square feet} \). The amount of area of the windows and the door is \( 2 \cdot 2 \cdot 4 + 4 \cdot 7 = 44 \text{ square feet} \). So, the area that needs to be covered is \( 384 - 44 = 340 \text{ square feet} \). You will need \( 340 \cdot 350 \approx 0.97 \) or about 1 gallon of paint to cover the walls with one coat of paint.

25. The approximate amount of glass needed for the lamp is \( 6 \cdot \frac{1}{2} \cdot 5 \cdot 7 = 105 \text{ square inches} \).

27. Sample answer: The student did not multiply by 2, and should review finding the surface area of a rectangular prism.

29. The surface area of the small cube is \( 6 \cdot 6^2 = 216 \text{ square inches} \) and the surface area of the large cube is \( 6 \cdot 8^2 = 384 \text{ square inches} \). So, the ratio of the surface area of the smaller cube to the surface area of the larger cube is \( \frac{216}{384} = \frac{9}{16} = 0.5625 \).

31. The surface area is multiplied by a factor of 4 since each of the sides of the box is multiplied by a factor of 4. So, you will need about \( 4 \cdot 135 = 540 \text{ square inches of cardboard} \) for the large box of pasta.

33. (a) There is no change in the surface area of the larger cube. Sample answer: The area removed equals the area added.
(b) The area of the faces of the larger cube is \( 96 \cdot 6 = 16 \text{ square inches} \). So, the edge length of the large cube is 4 inches. The area of the faces of the smaller cube is \( 6 + 6 = 1 \text{ square inch} \). So, the edge length of the smaller cube is 1 inch. The surface area of the new solid is:
\[
S = 3 \cdot 4 \cdot 4 + (3 \cdot 4 \cdot 4 - 3 \cdot 1 \cdot 1) + 3 \cdot 1 \cdot 1
= 48 + 45 + 3
= 96 \text{ in.}^2
\]
(c) The answer in part (a) was correct. Sample answer: The surface areas are equal.
35. 1. **Understand the Problem** Find the surface area of the new solid.
2. **Make a Plan** Find the surface area of the larger prism and subtract the surface area of the faces that the two prisms share. Then add the surface area of the remaining faces of the smaller prism.
3. **Solve the Problem**
   Surface area of the larger prism:
   \[ S = 2 \cdot 3 \cdot 3 + 4 \cdot 3 \cdot 8 = 18 + 96 = 114 \text{ ft}^2 \]
   Surface area of the shared faces:
   \[ S = 2 \cdot 0.5 \cdot 0.5 = 0.5 \text{ ft}^2 \]
   Surface area of the remaining sides of the smaller prism:
   \[ S = 4 \cdot 0.5 \cdot 8 = 16 \text{ ft}^2 \]
   The surface area of the new solid is \( 114 - 0.5 + 16 = 129.5 \text{ square feet} \).

37. **Sample answer:** The lateral surface area can be found by multiplying the height of the prism by the perimeter of the base.

39. (a) The length of the prism is 24 units, the width is 1 unit, and the height is 1 unit.
   \[ S = 2l \cdot w + 2l \cdot h + 2w \cdot h \]
   \[ = 2(24)(1) + 2(24)(1) + 2(1)(1) \]
   \[ = 48 + 48 + 2 \]
   \[ = 98 \]
   The surface area is 98 square units.
(b) **Sample answer:** To create a prism with a surface area of 70 square units, make the length 1 unit, the width 3 units, and the height 8 units.
(c) The prism with the smallest surface area is 4 units long, 3 units wide, and 2 units high.
   \[ S = 2l \cdot w + 2l \cdot h + 2w \cdot h \]
   \[ = 2(4)(3) + 2(4)(2) + 2(3)(2) \]
   \[ = 24 + 16 + 12 \]
   \[ = 52 \]
   The surface area is 52 square units.
(d) **Sample answer:** Students explore how prisms with the same volume can have different surface areas.

### Section 11.4 Volumes of Polyhedra

1. **Solution key:**
   1. \( 12 \text{ units}^3 \)
      \[ 12 \text{ units}^3 \]
      \[ + 12 \text{ units}^3 \]
      \[ 36 \text{ units}^3 \]
   2. \( 16 \text{ units}^3 \)
      \[ 16 \text{ units}^3 \]
      \[ 16 \text{ units}^3 \]
      \[ 16 \text{ units}^3 \]
      \[ + 16 \text{ units}^3 \]
      \[ 80 \text{ units}^3 \]
   3. \( 4 \text{ units}^3 \)
      \[ + 4 \text{ units}^3 \]
      \[ 8 \text{ units}^3 \]
   4. \( 6 \text{ units}^3 \)
      \[ 6 \text{ units}^3 \]
      \[ + 6 \text{ units}^3 \]
      \[ 18 \text{ units}^3 \]
   5. \( 8 \text{ units}^3 \)
      \[ 8 \text{ units}^3 \]
      \[ + 8 \text{ units}^3 \]
      \[ 24 \text{ units}^3 \]
   6. \( V = \ell \cdot w \)

**Sample rubric:** Assign 1 point to each correct volume or formula and 1 point to each correct prism. There is a total of 9 points.

3. The dimensions of the possible rectangular prisms are 1 by 1 by 8, 1 by 2 by 4, and 2 by 2 by 2. Each of the prisms have volumes of 8 cubic units.
4. \( 6 \text{ units}^3 \)
   \[ 6 \text{ units}^3 \]
   \[ 6 \text{ units}^3 \]
   \[ + 6 \text{ units}^3 \]
   \[ 24 \text{ units}^3 \]
   The volume of the prism is 24 cubic units.